where elementary identities have been used to convert the Bessel functions into a form suitable for integration. The dielectric conductivity $\sigma = \omega\epsilon''$, where $\epsilon = \epsilon' + \epsilon''$ is the complex permittivity and is related to the loss tangent by $\tan\delta = \epsilon''/\epsilon'$.

The cylindrical Green's function is well known (2);

$$\tfrac{1}{2}G_1 = \ln\frac{a}{r} - \sum \frac{1}{n}\left(\frac{r_0^n r^n}{a^{2n}} - \frac{r_0^n}{r^n}\right) \cdot \cos n(\phi_0 - \phi) \qquad r > r_0$$

$$\tfrac{1}{2}G_2 = \ln\frac{a}{r_0} - \sum \frac{1}{n}\left(\frac{r_0^n r^n}{a^{2n}} - \frac{r^n}{r_0^n}\right) \cdot \cos n(\phi_0 - \phi) \qquad r < r_0 \tag{4}$$

where r, ϕ refers to field points and r_0, ϕ_0 to source points. At $r = a$, $T = 0$. The heat flow equation may now be put in the form of the solution

$$T(r, \phi) = \frac{1}{2\pi\kappa}\int_0^{2\pi}\left[\int_0^r r_0 G_1 p(r_0, \phi_0)dr_0 + \int_r^a r_0 G_2 p(r_0, \phi_0)dr_0\right]d\phi_0. \tag{5}$$

Performing the indicated integrations and combining terms,

$$\begin{aligned}\frac{2\kappa}{\sigma E_0^2}T(r, \phi) &= r^2\left[\left(\frac{1}{k_c^2r^2} - 1\right)(J_0^2(k_cr) + J_1^2(k_cr)) + \frac{1}{2k_cr}J_0(k_cr)J_1(k_cr)\right] \\ &- a^2\left[\left(\frac{1}{k_c^2a^2} - 1\right)(J_0^2(k_ca) + J_1^2(k_ca)) + \frac{1}{2k_ca}J_0(k_ca)J_1(k_ca)\right] \\ &- \frac{r^2}{6}\left[\left(\frac{4}{k_c^2r^2} + 1\right)J_1^2(k_cr) - 2J_0^2(k_cr) + \frac{2}{k_cr}J_0(k_cr)J_1(k_cr) \right. \\ &\left. - \left(\frac{4}{k_c^2a^2} + 1\right)J_1^2(k_ca) + 2J_0^2(\kappa_ca) - \frac{2}{k_ca}J_0(k_ca)J_1(k_ca)\right]\cos 2\phi.\end{aligned} \tag{6}$$

The power flux, obtained by taking the time average of the Poynting vector over the cross section of the waveguide, may be shown to be

$$P = \pi E_0^2a^2J_1^2(k_ca)\left(1 - \frac{1}{k_c^2a^2}\right) \cdot \sqrt{1 - (\lambda/\lambda_c)^2}/\eta. \tag{7}$$

From this expression and the boundary condition for this mode, $k_ca = 1.84$, the central temperature may be determined:

$$T_0 = 0.4350\frac{\sigma\eta P}{\kappa\sqrt{1 - (\lambda/\lambda_c)^2}}\cdot \tag{8}$$

The temperature distribution is shown in Fig. 1.

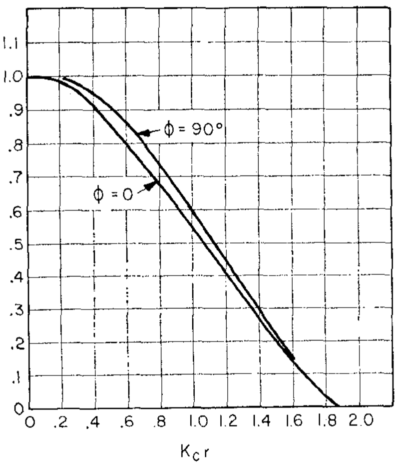

Fig. 1—Normalized temperature distribution for limiting cases.

ACKNOWLEDGMENT

The writer desires to acknowledge helpful discussions with Dr. R. V. Yadavalli during preparation of this note.

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A Vacuum Forming Technique for the Fabrication of Spherical or Prolate Spheroidal Reflectors*

A quick and inexpensive way of making spherical or prolate spheroidal reflectors for use in the millimeter wave range has been developed. This technique is based upon the fact that for small deflections a circular membrane stretched uniformly by a vacuum assumes approximately the shape of a spherical cap, and an elliptical membrane assumes approximately the shape of a prolate spheroidal cap. In making the reflectors, 0.001-inch aluminum foil is used as the membrane because of its low elastic limit. Once the foil is stretched, it retains its shape, eliminating the need for continual pumping.

A cross-sectional view of the apparatus used in making the reflectors is shown in Fig. 1. The vacuum table is $\frac{3}{8}$-inch thick aluminum and is fitted with a micrometer in the center. The aluminum foil is cemented to the reflector frame with an industrial adhesive, Eastman 910.[1] The reflector frame is insulated from the vacuum table so that the electrical connection made when the foil touches the micrometer post can be used to actuate a vacuum value to stop the forming process. This arrangement provides a means for accurately forming the foil to the correct depth.

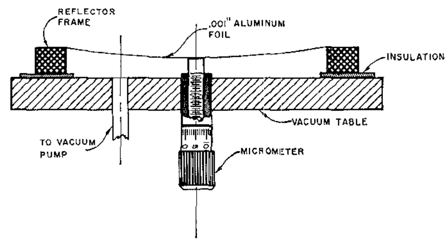

Fig. 1—Cross-sectional view of reflector fabrication apparatus.

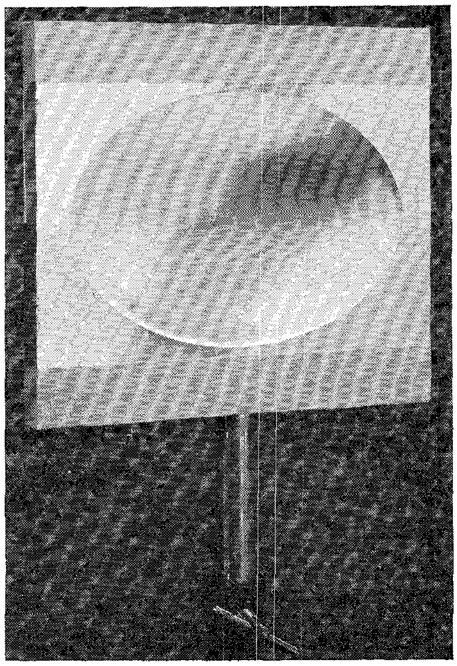

Fig. 2—Prolate spheroidal reflector.

The construction of the various parts is quite simple with the exception of the reflector frames when prolate spheroidal reflectors are being made. For this case elliptical holes must be cut in the frames, and to do this with a conventional milling machine requires that the frames be made in two sections. By tilting the vertical head of the mill at the correct angle, a fly cutter can be used to cut a semi-ellipse in each half of the frame and the two sections can then be rejoined using, for example, Eastman 910 cement. A completed prolate spheroidal reflector is shown in Fig. 2.

The curvature of the reflectors formed in this way was checked using templates and found to be sufficiently close to the calculated shape for use at the design wavelength of 4 mm. There tend to be small wrinkles at the edge of the foil due to uneven spreading of the adhesive, but these can be allowed for by making the reflectors slightly oversize. The depth of several different reflectors can easily be kept uniform to a tolerance of ±0.001-inch.

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* Received August 5, 1963. The research reported in this paper was sponsored by the U. S. Air Force, under RADC Contract AF 30(602)-3046 with the University of Illinois; Urbana, Ill.

[1] A product of Tennessee Eastman Co., Kingsport, Tenn.